Int J Theor Phys (2010) 49: 376–383
DOI 10.1007/s10773-009-0210-z

Controlled Dense Coding using Generalized GHZ-type State

Xiao-jie Yi · Jian-min Wang · Guo-qiang Huang

Received: 23 September 2009 / Accepted: 26 November 2009 / Published online: 8 December 2009

Abstract Two schemes for controlled dense coding with a extended GHZ-type state are investigated. In this protocol, the supervisor (Cliff) can control the channel and the average amount of information transmitted from the sender (Alice) to the receiver (Bob) by adjusting the local measurement angle θ. It is shown that the results for the average amounts of information are unique from the different two schemes.

Keywords Controlled dense coding · Extended GHZ-type state · POVM · Average amount of information

1 Introduction

Quantum entanglement, one of the most interesting features in quantum mechanics, plays a significant role in quantum mechanics, as it not only holds the power for demonstration of the quantum nonlocality against local hidden variable theory [1], but also provides promising and wide applications in quantum information processing, such as teleportation [2–7], dense coding [8, 9], quantum state sharing [10–14] and so on. Dense coding is one of the important branches of quantum information theory. In the original protocol, Bennett and Wiesner have showed how entangled states can increase the communication capacity of two parties. In an ideal classical channel, the transmission of 2 bits of information requires the manipulation and transmission of at least two particles or physical entities, which are used to encode the information. But if the two parties share a maximally bipartite entangled state, the sender can transmit 2 bits of information by manipulating and sending only one qubit [8, 9]. Since the original idea of quantum dense coding considered in 1992 by Bennett and Wiesner, dense

X.-j. Yi · J.-m. Wang
College of Communication and Electron, Jiangxi Science and Technology Normal University, Nanchang 330013, P.R. China

G.-q. Huang (✉)
Department of Electronics, Jiangxi University of Finance and Economics, Nanchang 330013, P.R. China
e-mail: guoqhuang@tom.com

coding has been generalized in various directions. For example, it is possible to generalize the dense coding for continuous variables [15, 16], multipartite communication [17–22]. One can perform dense coding not only with quantum states in finite dimensional Hilbert spaces but also with quantum states in infinite dimensional Hilbert spaces. Quantum dense coding was experimentally presented by Mattle et al. in an optical system [23], and then by Fang et al. by the use of NMR techniques [24].

The original controlled dense coding protocol was proposed in 2001 [25]. In this protocol, one party (Alice) can transmit information to the second party (Bob) whereas the local measurement of the third party (Cliff) serves as quantum erasure. Cliff can control the quantum channel between Alice and Bob and the average amount of information transmitted from Alice to Bob via a local measurement. It was experimentally demonstrated by Zhang and Jing for continuous variables [26, 27]. Later, Chen and Kuang generalized the controlled dense coding protocol of the three-particle GHZ quantum channel to the case of a $(N+2)$-particle GHZ quantum channel via a series of local measurements [28].

In this paper, two methods are shown to realize controlled dense coding with extended GHZ-type state [29, 30]. One of the strategies is Alice first concentrates the entanglement of the channel between herself and Bob, and then performs dense coding. The second one is Alice directly applies one of the four unitary operators $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$ on her qubit and then sends it to Bob. After receiving Alice's qubit, Bob can obtain 2 bits of information with a certain probability via performing a projective measurement and a generalized measurement described by positive-operator-valued measure (POVM) elements on his two qubit states [31].

2 Dense Coding with Entanglement Concentration

Let us assume that the quantum channel between the sender Alice, the receiver Bob and the supervisor Cliff is a extended GHZ-type state which is given by

$$|\Psi\rangle_{123} = \frac{1}{\sqrt{3}}(|000\rangle + |110\rangle + |111\rangle)_{123}, \tag{1}$$

where qubit 1 is hold by Alice, qubit 2 by Bob and qubit 3 by Cliff respectively. In order to control the quantum channel between Alice and Bob and the amount of information transmitted from Alice to Bob, Cliff performs a von Neumann measurement on his qubit 3 under the basis

$$|+\rangle_3 = \cos\theta|0\rangle_3 + \sin\theta|1\rangle_3, \qquad |-\rangle_3 = \sin\theta|0\rangle_3 - \cos\theta|1\rangle_3 \tag{2}$$

(θ is a measured angle with the region $0 \le \theta \le \pi/4$) and informs his measurement result to Alice and Bob through a classical channel. It is noted that the extended GHZ-type state in the new basis $\{|+\rangle_3, |-\rangle_3\}$ can be rewritten as

$$|\Psi\rangle_{123} = |\xi\rangle_{12} \otimes |+\rangle_3 + |\zeta\rangle_{12} \otimes |-\rangle_3, \tag{3}$$

where

$$\begin{aligned} |\xi\rangle_{12} &= \frac{1}{\sqrt{3}}(\cos\theta|01\rangle_{12} + \cos\theta|11\rangle_{12} + \sin\theta|11\rangle_{12}), \\ |\zeta\rangle_{12} &= \frac{1}{\sqrt{3}}(\sin\theta|00\rangle_{12} + \sin\theta|11\rangle_{12} - \cos\theta|11\rangle_{12}) \end{aligned} \tag{4}$$

are unnormalized state vectors and their norms stand for the absolute probabilities for each case. Obviously, corresponding to Cliff's measurement result $|+\rangle_3$ or $|-\rangle_3$, the state of qubits 1 and 2 collapses to $|\xi\rangle_{12}$ or $|\zeta\rangle_{12}$, respectively. Generally, the states $|\xi\rangle_{12}$ and $|\zeta\rangle_{12}$ are not maximally entangled, and the success probability of dense coding of 2 bits of information with them is less than 1.

Now let us analyze first the case in which Cliff's measurement gives $|+\rangle_3$ and the state of qubits 1 and 2 collapses to $|\xi\rangle_{12}$. After receiving the measurement result, Alice introduces an auxiliary qubit with original state $|0\rangle_{aux}$ and performs a unitary transformation

$$U_1 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\cos\theta}{\cos\theta+\sin\theta} & \sqrt{1-\frac{\cos^2\theta}{1+\sin 2\theta}} & 0 \\ 0 & \sqrt{1-\frac{\cos^2\theta}{1+\sin 2\theta}} & -\frac{\cos\theta}{\cos\theta+\sin\theta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \tag{5}$$

on the auxiliary qubit and qubit 1 under the basis $\{|0\rangle_{aux}|0\rangle_1, |0\rangle_{aux}|1\rangle_1, |1\rangle_{aux}|0\rangle_1, |1\rangle_{aux}|1\rangle_1\}$. The collective unitary transformation $U_1 \otimes I_2$ transforms the state $|0\rangle_{aux} \otimes |\xi\rangle_{12}$ to

$$\begin{aligned} |\xi\rangle_{aux12} = {} & \sqrt{\frac{2}{3}}\cos\theta|0\rangle_{aux} \otimes \left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] \\ & + \sqrt{\frac{1}{3}(\sin^2\theta + \sin 2\theta)}|1\rangle_{aux} \otimes |00\rangle_{12}. \end{aligned} \tag{6}$$

Then Alice performs a von Neumann measurement on the auxiliary qubit under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$ and informs Bob of the result. If she obtains $|0\rangle_{aux}$, qubits 1 and 2 are maximally entangled. Alice can perform one of the four unitary transformations $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$ on qubit 1 and send it to Bob. Then Bob knows he has two qubits in one of the four Bell states resulted from Alice's transformation. By performing a Bell-basis measurement, Bob can discriminate Alice's unitary transformation on qubit 1, so 2 bits of information are transmitted. If Alice obtains $|1\rangle_{aux}$, qubits 1 and 2 are unentangled. Bob can extract only 1 bit of information. So, on average

$$I_1^{|\xi\rangle_{12}} = \frac{1}{3} + \cos^2\theta + \frac{1}{3}\sin 2\theta \tag{7}$$

bits of information are transmitted from Alice to Bob.

If Cliff's measurement result is $|-\rangle_3$, the state of qubits 1 and 2 collapses to $|\zeta\rangle_{12}$. This situation is more complicated, Alice must perform corresponding unitary transformation according to the size of θ. First, we set the measurement angle satisfies $\arctan\frac{1}{2} \le \theta \le \frac{\pi}{4}$. Alice's unitary transformation on the auxiliary qubit and qubit 1 under the basis $\{|0\rangle_{aux}|0\rangle_1, |0\rangle_{aux}|1\rangle_1, |1\rangle_{aux}|0\rangle_1, |1\rangle_{aux}|1\rangle_1\}$ is

$$U_2 = \begin{pmatrix} \frac{\cos\theta-\sin\theta}{\sin\theta} & 0 & \sqrt{1-\frac{1-\sin 2\theta}{\sin^2\theta}} & 0 \\ 0 & -1 & 0 & 0 \\ \sqrt{1-\frac{1-\sin 2\theta}{\sin^2\theta}} & 0 & \frac{\sin\theta-\cos\theta}{\sin\theta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix}. \tag{8}$$

The collective unitary transformation $U_2 \otimes I_2$ transforms the state $|0\rangle_{aux} \otimes |\zeta\rangle_{12}$ to

$$|\zeta\rangle_{aux12} = \sqrt{\frac{2}{3}}(\cos\theta - \sin\theta)|0\rangle_{aux} \otimes \left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] + \sqrt{\frac{1}{3}(\sin 2\theta + \sin^2\theta - 1)}|1\rangle_{aux} \otimes |00\rangle_{12}. \quad (9)$$

Then Alice performs a von Neumann measurement on the auxiliary qubit under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$. If Alice gets the result $|1\rangle_{aux}$, the state of qubits 1, 2 is unentangled, and only 1 bit of information can be transmitted. If she gets $|0\rangle_{aux}$, the state of qubits 1 and 2 is maximally entangled, and 2 bits of information can be transmitted. So in the case, Alice can transmit

$$I_1^{|\zeta\rangle_{12}} = 1 + \frac{1}{3}\sin^2\theta - \sin 2\theta \quad (10)$$

bits of information on average.

The average amount of information transmitted from Alice to Bob adds up to

$$I = I_1^{|\xi\rangle_{12}} + I_1^{|\zeta\rangle_{12}} = \frac{5}{3} + \frac{2}{3}(\cos^2\theta - \sin 2\theta) \quad (11)$$

bits.

If $0 \le \theta \le \arctan\frac{1}{2}$, Alice's unitary transformation on the auxiliary qubit and qubit 1 under the same basis is

$$U_2 = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & \frac{\sin\theta}{\sin\theta - \cos\theta} & \sqrt{1 - \frac{\sin^2\theta}{1-\sin 2\theta}} & 0 \\ 0 & \sqrt{1 - \frac{\sin^2\theta}{1-\sin 2\theta}} & -\frac{\sin\theta}{\sin\theta-\cos\theta} & 0 \\ 0 & 0 & 0 & 1 \end{pmatrix} \quad (12)$$

and the state $|0\rangle_{aux} \otimes |\zeta\rangle_{12}$ is transformed to

$$|\zeta\rangle_{aux12} = \sqrt{\frac{2}{3}}\sin\theta|0\rangle_{aux} \otimes \left[\frac{1}{\sqrt{2}}(|00\rangle_{12} + |11\rangle_{12})\right] + \sqrt{\frac{1}{3}(\cos^2\theta - \sin 2\theta)}|1\rangle_{aux} \otimes |00\rangle_{12}. \quad (13)$$

After Alice's von Neumann measurement under the basis $\{|0\rangle_{aux}, |1\rangle_{aux}\}$, Alice can transmit

$$I_2^{|\zeta\rangle_{12}} = \frac{1}{3} + \sin^2\theta - \frac{1}{3}\sin 2\theta \quad (14)$$

bits of information on average. In this case, the average amount of information transmitted from Alice to Bob adds up to

$$I = I_1^{|\xi\rangle_{12}} + I_2^{|\zeta\rangle_{12}} = \frac{5}{3}. \quad (15)$$

Synthesizing two measurement cases,the average amount of information transmitted from Alice to Bob is summarized as

$$I = \begin{cases} I_1^{|\xi\rangle_{12}} + I_2^{|\zeta\rangle_{12}} = \frac{5}{3}, & 0 \le \theta \le \arctan\frac{1}{2}, \\ I_+^{|\xi\rangle_{12}} I_2^{|\zeta\rangle_{12}} = \frac{5}{3} + \frac{2}{3}(\cos^2\theta - \sin 2\theta), & \arctan\frac{1}{2} \le \theta \le \frac{\pi}{4}. \end{cases} \tag{16}$$

From (7), (10) and (14) we can see that the average amount of information transmitted from Alice to Bob not only depends on Cliff's measurement result $|\pm\rangle_3$ but also on the measurement angle θ, that is to say, by adjusting the value of θ, Cliff can control the transmission from Alice to Bob, which is also the control of the entanglement of the channel between Alice and Bob.

3 Controlled Dense Coding via Generalized Measurement

In this section, we first consider the case of Cliff's measurement is $|+\rangle_3$, and the other case can be deduced later. After receiving the measurement result, Alice doesn't concentrate the channel, she directly uses any one of the four unitary operators $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$ to operate the shared state $|\xi\rangle_{12}$. Depending on the applied unitary transformation, the shared state $|\xi\rangle_{12}$ undergoes one of the following transformations:

$$\begin{aligned} (I \otimes I)|\xi\rangle_{12} &= \frac{1}{\sqrt{3}}[\cos\theta|00\rangle_{12} + (\cos\theta + \sin\theta)|11\rangle_{12}] = |\phi_1\rangle_{23}, \\ (\sigma_X \otimes I)|\xi\rangle_{12} &= \frac{1}{\sqrt{3}}[\cos\theta|10\rangle_{12} + (\cos\theta + \sin\theta)|01\rangle_{12}] = |\phi_2\rangle_{12}, \\ (i\sigma_Y \otimes I)|\xi\rangle_{12} &= \frac{1}{\sqrt{3}}[-\cos\theta|10\rangle_{12} + (\cos\theta + \sin\theta)|01\rangle_{12}] = |\phi_3\rangle_{12}, \\ (\sigma_Z \otimes I)|\xi\rangle_{12} &= \frac{1}{\sqrt{3}}[\cos\theta|00\rangle_{12} - (\cos\theta + \sin\theta)|11\rangle_{12}] = |\phi_4\rangle_{12}. \end{aligned} \tag{17}$$

Then Alice sends qubit 1 to Bob, and now Bob has at his disposal two qubits which could be in any one of the four possible states $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$. However, the above four states are not mutually orthogonal. According to quantum theory, it is obvious that these four non-orthogonal states cannot be distinguished with certainty. But it is known that a set of non-orthogonal states which are linearly independent can be distinguished with some probability of success [32–34]. In fact, it is easy to find that the above set $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$ is actually linearly independent. Therefore Bob can conclusively distinguish these states with some probability of success.

To distinguish the above set $\{|\phi_1\rangle_{12}, |\phi_2\rangle_{12}, |\phi_3\rangle_{12}, |\phi_4\rangle_{12}\}$, first Bob performs a projection onto the subspaces spanned by the basis states $\{|00\rangle, |11\rangle\}$ and $\{|01\rangle, |10\rangle\}$ with corresponding projective operators are $P_1 = |00\rangle\langle 00| + |11\rangle\langle 11|$ and $P_2 = |01\rangle\langle 01| + |10\rangle\langle 10|$ respectively. Obviously, P_1 and P_2 are mutually orthogonal, and Bob can discriminate the two subsets of Alice's operators: $\{I, \sigma_Z\}$ and $\{\sigma_X, i\sigma_Y\}$. If Bob obtains P_1, then he knows that the state will be either $|\phi_1\rangle_{12}$ or $|\phi_4\rangle_{12}$. Similarly, if he obtains P_2, the state will be either $|\phi_2\rangle_{12}$ or $|\phi_3\rangle_{12}$. After this projective measurement he gets 1 bit of information [35]. Now we suppose Bob obtains P_1, then he performs a generalized measurement on his two qubit states. In the case, the corresponding positive operator valued measure (POVM) elements in the subspace $\{|00\rangle, |11\rangle\}$ are [31, 35]

$$
\begin{aligned}
M_1 &= \frac{1}{2}\begin{pmatrix} \frac{\cos^2\theta}{1+\sin 2\theta} & \frac{\cos\theta}{\cos\theta+\sin\theta} \\ \frac{\cos\theta}{\cos\theta+\sin\theta} & 1 \end{pmatrix}, \\
M_2 &= \frac{1}{2}\begin{pmatrix} \frac{\cos^2\theta}{1+\sin 2\theta} & -\frac{\cos\theta}{\cos\theta+\sin\theta} \\ -\frac{\cos\theta}{\cos\theta+\sin\theta} & 1 \end{pmatrix}, \\
M_3 &= \begin{pmatrix} \frac{\sin^2\theta+\sin 2\theta}{1+\sin 2\theta} & 0 \\ 0 & 0 \end{pmatrix}.
\end{aligned}
\tag{18}
$$

It is easy to check that the condition $M_1 + M_2 + M_3 = I$ is satisfied.

The generalized measurement has three outcomes. If Bob gets M_1 then the state is $|\phi_1\rangle_{12}$, if he gets M_2 then the state is $|\phi_4\rangle_{12}$. However if he gets M_3 the state is completely indecisive and Bob cannot obtain any information. The success probability of distinguishing $|\phi_1\rangle_{12}$ and $|\phi_4\rangle_{12}$ is $\frac{2\cos^2\theta}{1+\sin 2\theta+\cos^2\theta}$, which is also the probability that Bob obtains another 1 bit of information. Similar procedure can be applied for the case of P_2, one can easily check that the relevant POVM elements and the success probability are the same. So, in this case, the average amount of information transmitted from Alice to Bob should be expressed as

$$
\begin{aligned}
I_{1'}^{|\xi\rangle_{12}} &= \frac{1}{3}(1+\sin 2\theta+\cos^2\theta)\times 1 + \frac{1}{3}(1+\sin 2\theta+\cos^2\theta)\times\frac{2\cos^2\theta}{1+\sin 2\theta+\cos^2\theta} \\
&= \frac{1}{3}(1+\sin 2\theta+\cos^2\theta) + \frac{2}{3}\cos^2\theta.
\end{aligned}
\tag{19}
$$

If Charlies measurement result is $|-\rangle_3$, then the state of qubits 1 and 2 collapses to $|\zeta\rangle_{12}$. After Alice encoding with one of the four operators $\{I, \sigma_X, i\sigma_Y, \sigma_Z\}$, the state $|\zeta\rangle_{12}$ undergoes one of the following transformations:

$$
\begin{aligned}
(I\otimes I)|\zeta\rangle_{12} &= \frac{1}{\sqrt{3}}(\sin\theta|00\rangle_{12} + \sin\theta|11\rangle_{12} - \cos\theta|11\rangle_{12}) = |\varphi_1\rangle_{12}, \\
(\sigma_X\otimes I)|\zeta\rangle_{12} &= \frac{1}{\sqrt{3}}(\sin\theta|01\rangle_{12} - \cos\theta|01\rangle_{12} + \sin\theta|10\rangle_{12}) = |\varphi_2\rangle_{12}, \\
(i\sigma_Y\otimes I)|\zeta\rangle_{12} &= \frac{1}{\sqrt{3}}(\sin\theta|01\rangle_{12} - \cos\theta|01\rangle_{12} - \sin\theta|10\rangle_{12}) = |\varphi_3\rangle_{12}, \\
(\sigma_Z\otimes I)|\zeta\rangle_{12} &= \frac{1}{\sqrt{3}}(\sin\theta|00\rangle_{12} - \sin\theta|11\rangle_{12} + \cos\theta|11\rangle_{12}) = |\varphi_4\rangle_{12}.
\end{aligned}
\tag{20}
$$

After receiving the qubit, Bob first performs a projection onto the subspaces spanned by the basis states $|00\rangle, |11\rangle$ and $|01\rangle, |10\rangle$ and then a generalized measurement. In this case, the projection measurement is the same as earlier, but the POVM set depends on the relative size of $\sin\theta$ and $\cos\theta - \sin\theta$. So this situation can be divided into two ways to discuss. When

$0 \le \theta \le \arctan\frac{1}{2}$, the POVM set is

$$M_1' = \frac{1}{2}\begin{pmatrix} \frac{\sin^2\theta}{1-\cos 2\theta} & \frac{\sin\theta}{\cos\theta-\sin\theta} \\ \frac{\sin\theta}{\cos\theta-\sin\theta} & 1 \end{pmatrix},$$

$$M_2' = \frac{1}{2}\begin{pmatrix} \frac{\sin^2\theta}{1-\cos 2\theta} & -\frac{\sin\theta}{\cos\theta-\sin\theta} \\ -\frac{\sin\theta}{\cos\theta-\sin\theta} & 1 \end{pmatrix}, \tag{21}$$

$$M_3' = \begin{pmatrix} \frac{\cos^2\theta-\sin 2\theta}{1-\sin 2\theta} & 0 \\ 0 & 0 \end{pmatrix}.$$

In this case, Bob can discriminate $|\varphi_1\rangle_{12}$ from $|\varphi_4\rangle_{12}$ (or $|\varphi_2\rangle_{12}$ from $|\varphi_3\rangle_{12}$) with probability $\frac{2\sin^2\theta}{1-\sin 2\theta+\sin^2\theta}$. That is to say, Bob can obtain

$$\begin{aligned} I_{2'}^{|\zeta\rangle_{12}} &= \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)\times 1 + \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)\times\frac{2\sin^2\theta}{1-\sin 2\theta+\sin^2\theta} \\ &= \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)+\frac{2}{3}\sin^2\theta \end{aligned} \tag{22}$$

bits of information from Alice.

When $\arctan\frac{1}{2} \le \theta \le \frac{\pi}{4}$, the POVM set is

$$M_1'' = \frac{1}{2}\begin{pmatrix} \frac{1-\sin 2\theta}{\sin^2\theta} & \frac{\cos\theta-\sin\theta}{\sin^2\theta} \\ \frac{\cos\theta-\sin\theta}{\sin^2\theta} & 1 \end{pmatrix},$$

$$M_2'' = \frac{1}{2}\begin{pmatrix} \frac{1-\sin 2\theta}{\sin^2\theta} & -\frac{\cos\theta-\sin\theta}{\sin^2\theta} \\ -\frac{\cos\theta-\sin\theta}{\sin^2\theta} & 1 \end{pmatrix}, \tag{23}$$

$$M_3'' = \begin{pmatrix} \frac{\cos^2\theta-\sin 2\theta}{\sin^2\theta} & 0 \\ 0 & 0 \end{pmatrix}$$

and Alice can transmit

$$\begin{aligned} I_{2''}^{|\zeta\rangle_{12}} &= \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)\times 1 + \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)\times\frac{2(1-\sin 2\theta)}{1-\sin 2\theta+\sin^2\theta} \\ &= \frac{1}{3}(1-\sin 2\theta+\sin^2\theta)+\frac{2}{3}(1-\sin 2\theta). \end{aligned} \tag{24}$$

Synthesizing all measurement cases, the average amount of information transmitted from Alice to Bob can be expressed as

$$I = \begin{cases} I_{1'}^{|\xi\rangle_{12}} + I_{2'}^{|\zeta\rangle_{12}} = \frac{5}{3}, & 0 \le \theta \le \arctan\frac{1}{2}, \\ I_{1'}^{|\xi\rangle_{12}} + I_{2''}^{|\zeta\rangle_{12}} = \frac{5}{3} + \frac{2}{3}(\cos^2\theta - \sin 2\theta), & \arctan\frac{1}{2} \le \theta \le \frac{\pi}{4}. \end{cases} \tag{25}$$

Comparing (16) with (25), we find that the results are same, which means that the two schemes are equivalent for the controlled dense coding and heir results are unique. In addition, the second scheme needs less physical resources because it does not require the introduction of auxiliary particle, which is to its advantage.

4 Summary

In summary, two schemes, via entanglement concentration and generalized measurement respectively, of realizing controlled dense coding are investigated with a extended GHZ-type state. It is shown that the average amount of information transmitted from Alice to Bob only depends on Cliff's measured angle θ, which implies that Cliff can control the average amount of information by only adjusting the measured angle θ on his qubit. It is also shown that the results for the average amounts of information are unique from the different two schemes.

References

1. Einstein, A., Podolsky, B., Rosen, N.: Phys. Rev. **47**, 777 (1935)
2. Bennett, C.H., Brassard, G., Crepeau, C., Jozsa, R., Peres, A., Wooters, W.K.: Phys. Rev. Lett. **70**, 1895 (1993)
3. Zhang, Z.J., Liu, Y.M., Man, Z.X.: Commun. Theor. Phys. **44**, 847 (2005)
4. Zhang, Z.J.: Phys. Lett. A **352**, 55 (2006)
5. Zhang, Z.J., Man, Z.X.: Phys. Lett. A **341**, 55 (2005)
6. Han, L.F., Fang, M., Shi, S.H., Zhang, Z.J.: Commun. Theor. Phys. **46**, 217 (2006)
7. Wang, D., Liu, Y.M., Gao, G., Shi, S.H., Zhang, Z.J.: Commun. Theor. Phys. **47**, 437 (2007)
8. Bennett, C.H., Wiesner, S.J.: Phys. Rev. Lett. **69**, 2881 (1992)
9. Liu, J., Zhang, Z.J.: Commun. Theor. Phys. **49**, 887 (2008)
10. Wang, Z.Y., Liu, Y.M., Wang, D., Zhang, Z.J.: Opt. Commun. **276**, 322 (2007)
11. Zhang, Z.J., et al.: Phys. Rev. A **71**, 044301 (2005)
12. Zhang, Z.J., et al.: Phys. Rev. A **72**, 022303 (2005)
13. Zhang, Z.J., et al.: Opt. Commun. **269**, 418 (2007)
14. Wang, Z.Y., Liu, Y.M., Wang, D., Zhang, Z.J.: Opt. Commun. **276**, 322 (2007)
15. Braunstein, S.L., Kimble, H.J.: Phys. Rev. A **61**, 042302 (2000)
16. Ban, M.: Opt. Commun. **189**, 97 (2001)
17. Liu, X.S., et al.: Phys. Rev. A **65**, 022304 (2002)
18. Grudka, A., Wojcik, A.: Phys. Rev. A **66**, 014301 (2002)
19. Zhang, Z.J., Man, Z.X., Li, Y.: Int. J. Quantum Inf. **2**, 521 (2004)
20. Zhang, Z.J., Man, Z.X., Li, Y.: Phys. Lett. A **333**, 46 (2004)
21. Zhang, Z.J., Man, Z.X., Li, Y.: Chin. Phys. Lett. **22**, 18 (2005)
22. Man, Z.X., Zhang, Z.J., Li, Y.: Chin. Phys. Lett. **22**, 22 (2005)
23. Mattle, K., et al.: Phys. Rev. Lett. **76**, 4656 (1996)
24. Fang, X., et al.: Phys. Rev. A **61**, 022307 (2002)
25. Hao, J.C., et al.: Phys. Rev. A **63**, 054301 (2001)
26. Zhang, J., et al.: Phys. Rev. A **66**, 032318 (2002)
27. Jing, J.T., et al.: Phys. Rev. Lett. **90**, 167903 (2003)
28. Chen, J.L., Kuang, L.M.: Chin. Phys. Lett. **21**, 12 (2004)
29. Acin, A., Brub, D., Lewenstein, M., Sanpera, A.: Phys. Rev. Lett. **87**, 040401 (2001)
30. Sim, S., Jin, J., Kwon, Y.: Int. J. Theor. Phys. **44**, 1419 (2005)
31. Pati, A.K., et al.: Phys. Rev. A **72**, 012329 (2005)
32. Duan, L.M., Guo, G.C.: Phys. Rev. Lett. **80**, 4999 (1998)
33. Chefles, A.: Contemp. Phys. **41**, 401 (2000)
34. Peres, A.: Phys. Lett. A **128**, 19 (1988)
35. Mor, T., Horodecki, P.: arXiv:quant-ph/9906039